Colourings of cubic graphs inducing isomorphic monochromatic subgraphs

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BAN – LINIAL CONJECTURE

WORMALD CONJECTURE

ANDO CONJECTURE

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WHAT ARE RELATIONS BETWEEN THEM?

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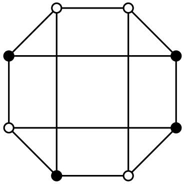

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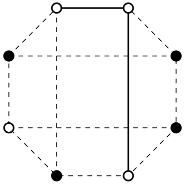

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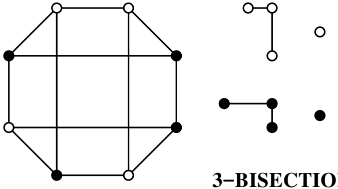
- We refer to a connected component of the subgraphs induced by a colour class as a *monochromatic component*.
- A *k–bisection* of a graph G is a 2–colouring c of the vertex set $V(G)$ such that:
  (i) $|\mathcal{B}| = |\mathcal{W}|$ (i.e. it is a bisection), and
  (ii) each monochromatic component has at most k vertices.



3–BISECTION

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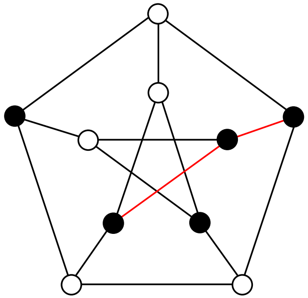
Conjecture (Ban–Linial; 2016)

Every bridgeless cubic graph admits a 2–bisection, except for the Petersen graph.

An exception: Petersen Graph

The Petersen graph has no 2-bisection.



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Let G be a cubic graph with $|E(G)| \equiv 0 \mod 2$ (or equivalently $|V(G)| \equiv 0 \mod 4$). Then there exists a linear partition of G in two isomorphic linear forests.

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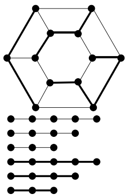

Main contents

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- Computational and theoretical evidence in their support.
- Open problems stronger than the above mentioned conjectures.

Ban–Linial's Conjecture: known results

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- Ban-Linial conjecture is open for BRIDGELESS cubic graphs.
- (Esperet, Mazzuoccolo and Tarsi, 2016): construction of an infinite class of 1-connected cubic graphs without a 2-bisection.
- The best general result is the following:

Proposition (L.Esperet, G.Mazzuoccolo, M.Tarsi, 2016)

Every cubic graph has a 3-bisection

Ban–Linial's Conjecture: New Results

- (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017): Ban–Linial's Conjecture evidences (2–bisections of cycle permutation and claw–free graphs)

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- (Abreu, Goedgebeur, DL, Mazzuoccolo, 2017): There is no bridgeless cubic graph of order up to 32 without a 2-bisection, but the Petersen.

Ban–Linial Conjecture: (Computational) New results

- (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017): There are exactly 34 graphs among the cubic graphs with at most 32 vertices which do not admit a 2–bisection. All of these graphs, except the Petersen graph, have connectivity 1.

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Order	# no 2–bisection
$0-8$	0
10	1
$12-20$	0
22	1
24	1
26	3
28	5
30	9
32	14

Table: Number of cubic graphs which do not admit a 2–bisection.

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- Hence this provides further evidence to support the correctness of Ban–Linial's Conjecture.

Isomorphic induced monochromatic subgraphs

REMARK:

In every 2-bisection, the two induced monochromatic subgraphs are isomorphic.

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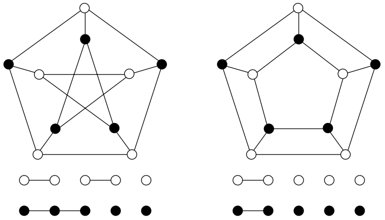
This is not the case, in general, for a k-bisection with $k > 2$.



Ando's Conjecture

Conjecture (Ando's Conjecture - 90's)

Every cubic graph admits a bisection such that the two induced monochromatic subgraphs are isomorphic.

Important remarks:

- If a cubic graph admits a 2-bisection then it is NOT a counterexample for Ando's Conjecture,
- BUT Ando's Conjecture is not restricted to bridgeless cubic graphs such as Ban-Linial's Conjecture.

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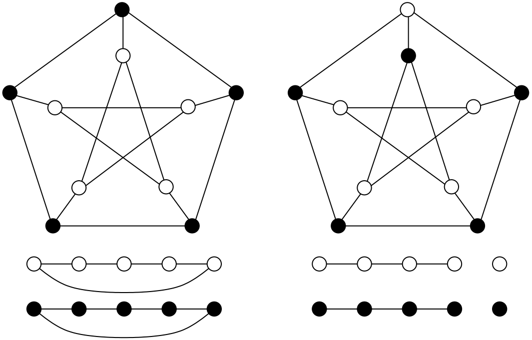
Proposition (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017)

Let G be a bridgeless cubic graph. If G is a counterexample for Ando's Conjecture, then it is also a counterexample for the Ban–Linial Conjecture.

Ando's Conjecture: Cubic graphs without a 2-bisection

Petersen graph is not a counterexample for Ando's Conjecture:



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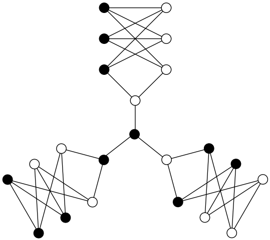

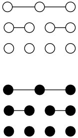

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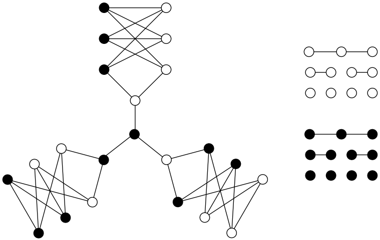

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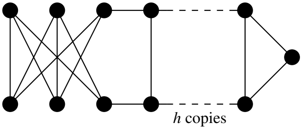

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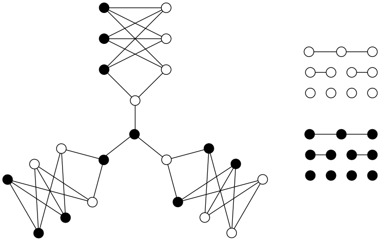

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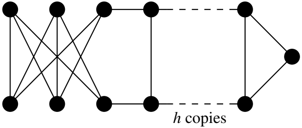

Proposition (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017)

All members of this family of 1–connected cubic graphs with no 2–bisection admit a 3–bisection with isomorphic parts.

We propose the following stronger version of Ando's Conjecture:

Conjecture (Strong Ando's Conjecture)

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- All graphs of order at most 24 and all known graphs without a 2–bisection are not counterexamples!

Strong Ando's Conjecture: Computational Results

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- All 34 cubic graphs which do not admit a 2–bisection up to 32 vertices, except the Petersen graph, have a 3–bisection such that the two isomorphic induced monochromatic graphs are linear forests.

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Corollary (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017)

The Strong version of Ando's conjecture (and thus also Ando's original conjecture) does not have any counterexamples with less than 34 vertices.

Synopsis

Conjecture Name	Colouring	Monochromatic components	Additional properties	Subclass Cubic Graphs
Ban-Linial	Vertices	Paths ≤ 2 vertices	Isomorphic parts	Bridgeless*
Ando	Vertices	No restriction	Isomorphic parts	ALL
Wormald	Edges	Paths	Isomorphic parts	$\|V\| \equiv 0 \pmod 4$
Strong Ando	Vertices	Paths	Isomorphic parts	ALL

* except Petersen Graph

Strong Wormald colourings vs Strong Ando colourings

Definition

Let G be a cubic graph. An Ando Colouring is a vertex colouring $c_V : V(G) \longrightarrow \{B, W\}$ of G such that the monochromatic induced subgraphs are isomorphic .

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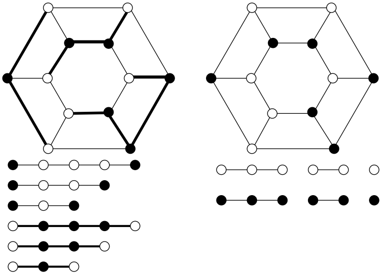

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- The answer is negative in general.
- (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017): construct an infinite family of graphs without a Strong Wormald Colouring.
- GOOD NEWS: the family of these examples seems to represent a very thin subclass of the class of all cubic graphs and, even if they do not admit a Strong Wormald Colouring, all of them admit a 2–bisection and hence a Strong Ando Colouring.

Cubic Graphs with no Strong Wormald Colouring

- The next natural step is the search of cubic graphs (of order a multiple of four) without a Strong Wormald Colouring.

Cubic Graphs with no Strong Wormald Colouring

- We have completed an exhaustive search for cubic graphs without a Strong Wormald Colouring up to 28 vertices:

Cubic Graphs with no Strong Wormald Colouring

- (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017): There are exactly 131 graphs without a Strong Wormald Colouring among the cubic graphs of order congruent to 0 (mod 4) and at most 28 vertices.

Order	Conn. 1	Conn. 2	Conn. 3	Total
4	0	0	0	0
8	0	0	0	0
12	0	0	0	0
16	1	1	1	3
20	20	3	1	24
24	18	1	0	19
28	72	12	1	85

Table: Number of cubic graphs of order 0 (mod 4) up to 28 vertices which do not admit a strong Wormald Colouring.

Cubic Graphs with no Strong Wormald Colouring

- Moreover, we have also computationally verified among the 131 examples that:

Corollary (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017)

Wormald's Conjecture does not have any counterexamples with less than 32 vertices.

Cubic Graphs with no Strong Wormald Colouring

- Note that all 2-connected examples admit a 3–edge colouring and hence they admit a 2–bisection and an Ando Colouring as well.

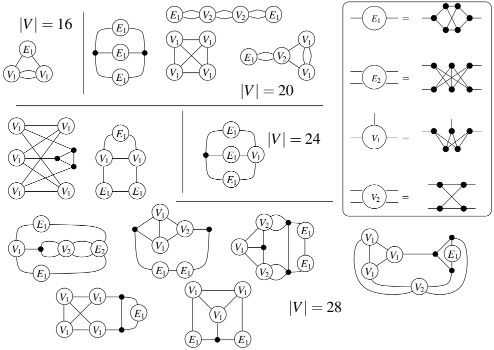

Figure: A complete list of all 2–connected cubic graphs without a Strong Wormald Colouring of order at most 28

Cubic Graphs with no Strong Wormald Colouring

- (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017): an infinite family of graphs without a Strong Wormald Colouring admitting a 2–bisection.

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- (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017): an infinite family of graphs without a Strong Wormald Colouring admitting a 2–bisection.
- Hence the infinite family is not a counterexample for Ando's Conjecture.

Strong Wormald Colouring of cubic graphs: $|V| \equiv 2 \pmod 4$

Facts

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Facts

- Evident connection between Wormald and (Strong) Ando's Conjectures, but also a large gap between them.
- Indeed, Wormald's Conjecture is only for cubic graphs of order a multiple of four while (Strong) Ando's Conjecture is stated for all cubic graphs.

Strong Wormald Col. of cubic graphs: $|V| \equiv 2 \pmod 4$

Recall that

Proposition (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017)

Let G be a cubic graph graph with $|V(G)| \equiv 2 \mod 4$ admitting a Strong Wormald Colouring. Then, G admits a Strong Ando Colouring (and so an Ando Colouring).

Strong Wormald Col. of cubic graphs: $|V| \equiv 2 \pmod 4$

- (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017) – Computationally: Every cubic graph of order 2 (mod 4) and at most 22 vertices admits a Strong Wormald Colouring (and hence a (Strong) Ando Colouring).

Strong Wormald Col. of cubic graphs: $|V| \equiv 2 \pmod 4$

- (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017) – Computationally: Every cubic graph of order $2 \pmod 4$ and at most 22 vertices admits a Strong Wormald Colouring (and hence a (Strong) Ando Colouring).

Problem (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017)

Does every cubic graph of order $2 \pmod 4$ *have a Strong Wormald Colouring?*

Strong Wormald Col. of cubic graphs: $|V| \equiv 2 \pmod 4$

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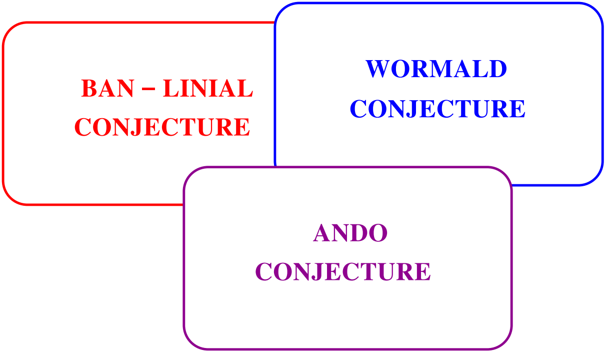
Problem (Abreu, Goedgebeur, DL, Mazzuoccolo; 2017)

Does every cubic graph of order $2 \pmod 4$ *have a Strong Wormald Colouring?*

- Proposition implies that a positive answer to Problem would imply (Strong) Ando's conjecture for all cubic graphs of order order $2 \pmod 4$.

Conclusion: a promising strategy to attack Ando's Conjecture



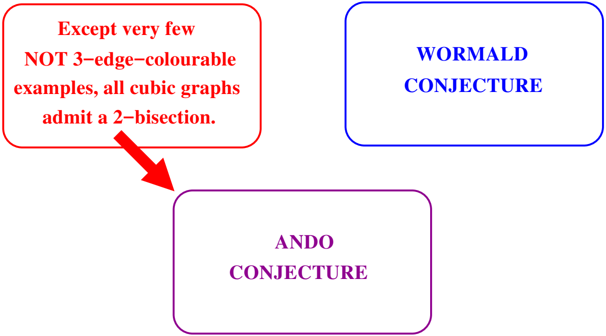
Conclusion: a promising strategy to attack Ando's Conjecture

BAN − LINIAL CONJECTURE

WORMALD CONJECTURE

ANDO CONJECTURE

Conclusion: a promising strategy to attack Ando's Conjecture



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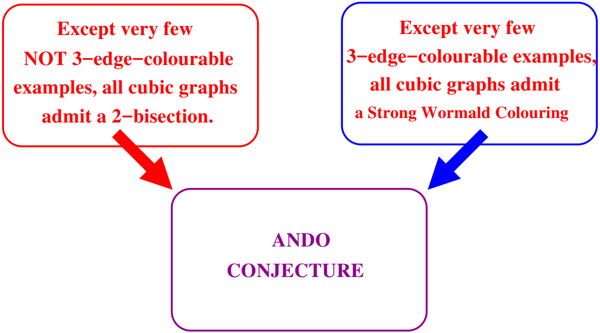

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Except very few
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all cubic graphs admit
a Strong Wormald Colouring

Ando Conjecture has
no known counterexample

THANKS FOR YOUR ATTENTION