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We call a graph *polyhedral* if it is planar and 3-connected. Let c_k be the order of the smallest k-regular polyhedral graph that is non-hamiltonian. By Euler's formula, $k \in \{3, 4, 5\}$. After a long series of papers by various authors improving the bounds for c_3, Holton and McKay [1] were able to solve the cubic case and show that the famous Lederberg-Bosák-Barnette graph on 38 vertices is indeed one of the (in total six) smallest 3-regular polyhedral graphs that are non-hamiltonian—thus, we have $c_3 = 38$.

However, far less is known about c_4 and c_5. In ongoing work, and dramatically improving previous bounds, Nico Van Cleemput and I have shown that $35 \leq c_4 \leq 39$. The currently best known lower and upper bound for the 5-regular case are 38 and 76, respectively. The former was proven running a computer program (with little optimisation), while the latter was shown by Owens [2].

My main question is for improving the bounds for c_4 and c_5. A secondary question revolves around determining p_k, which is the order of the smallest k-regular polyhedral graph that is non-traceable (i.e. does not contain a hamiltonian path). Various authors have shown that $54 \leq p_3 \leq 88$, $34 \leq p_4 \leq 78$, and $38 \leq p_5 \leq 120$, which are the best currently available bounds. As for the non-hamiltonian case, any improvements of these bounds would be very welcome.

References

[1] D. A. Holton and B. D. McKay. The smallest non-Hamiltonian 3-connected cubic planar graphs have 38 vertices. *J. Combin. Theory, Ser. B* **45** (1988) 305–319.

[2] P. J. Owens. On regular graphs and Hamiltonian circuits, including answers to some questions of Joseph Zaks. *J. Combin. Theory, Ser. B* **28** (1980) 262–277.