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Let r be a positive integer. An r-graph is an r-regular multigraph G such that for every odd set X of vertices of G there are at least r edges with precisely one end in X. Clearly, each r-graph has a 1-factor.

Conjecture. *If G is a planar r-graph, then it has two 1-factors F_1, F_2 with $F_1 \cap F_2 = \emptyset$.*

This conjecture is true if Seymour's conjecture that every planar r-graph is r-edge-colorable (see e.g. Chudnovsky M., Edwards, K., Seymour, P., Edge-colouring eight-regular planar graphs, J. Combin. Theory Ser. B 115 (2015) 303–338) is true. Seymour's conjecture is proved to be true if $r \leq 8$.

Rizzi (Rizzi, R., Indecomposable r-graphs and some other counterexamples, J. Graph Theory 32 (1999) 1–15) constructed r-graphs with the property that any two 1-factors of these graphs intersect (as it is always the case if $r = 3$). Such graphs are called poorly matchable. Hence the conjecture states that there is no planar poorly matchable r-graph. For $r = 3$ this is the 4-color theorem.