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The *cyclability* $\mathrm{cyc}(G)$ of a 2-connected graph G is the greatest r such that for every set $S \subseteq V(G)$ with $|S| \leq r$ there is a cycle in G which includes S. Note that $\mathrm{cyc}(G) = \infty$ if G is hamiltonian—a case we are not concerned with.

A general survey of cyclability was published by Gould [2]. On the present occasion we will consider only the case of k-regular k-connected graphs. Define $f(k)$ to be the least m such that $\mathrm{cyc}(G) \geq m$ for every k-regular k-connected graph G.

The only exactly known value of $f(k)$ for $k \geq 3$ is $f(3) = 9$ [5]. In the general case, Holton [4] proved that $f(k) \geq k + 4$, while Häggkvist and Mader [3] proved that $f(k) \geq k + \lfloor \frac{1}{3}\sqrt{k} \rfloor$. In the other direction, McCuaig and Rosenfeld [6] proved that $f(k) \leq 6k - 4$ if $k \equiv 0 \pmod 4$ and $f(k) \leq 8k - 5$ if $k \equiv 2 \pmod 4$.

The problem here is: *what is $f(k)$ for $k \geq 4$?* As warm-ups, we can ask for a better lower bound on $f(4)$ (the current lower bound of 8 being less than $f(3)$), and for proof or disproof that there is some constant $c > 1$ such that $f(k) \geq ck$ when k is large enough.

References

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